INSTABILITY OF FLEXURAL VIBRATIONS OF PLATES IN A TURBULENT BOUNDARY LAYER

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A large number of papers on the flutter of plates in supersonic flows has now been published (see, for example, [1-3]). The instability of elastic vibrations of a plate in a boundary layer of a substantially subsonic flow have been analyzed for disturbances in the form of harmonic traveling waves [4, 5]. The instability of periodic deflection of an infinite chain of plates in a turbulent boundary layer of an incompressible flow was studied in [6].

In the present paper we study the instability of flexural vibrations of bounded thin plates (panels) located beneath a turbulent boundary layer at the same level as a rigid flat surface. The Mach number of the flow is assumed to be small. Single-mode vibrations of a single rectangular plate and pairs of adjacent small plates hinged at the edges are studied.

The problem of the instability of flexural vibrations of plates in boundary layers is of interest, in particular, in connection with the problem of suppressing panel flutter. The appearance of instability – spontaneous growth of flexural vibrations due to energy provided by the flow – can result in more intense vibrations of the surface than would occur when the surface is excited passively by turbulent pulsations of the pressure [2, 7, 8]. The study of instability "in the small" makes it possible to proceed to the study of steady vibrations. Analysis of the interaction of adjacent small plates can serve as a foundation for the description of vibrations in long chains of plates, modeling a large panel surface.

1. Response of Flow to Single-Mode Harmonic Vibrations of a Small Rectangular Plate. We shall study the vibrations of a small plate, located at the same level as a flat rigid surface $y = 0$, and a plane-parallel flow of liquid with density ρ in the half-space $y > 0$. The density of the medium in the region $y < 0$ is assumed to be negligibly low. The flow-velocity profile $\bar{u}(y)$ is identical to the profile of the longitudinal velocity of the average flow in the turbulent layer and reaches a constant level $\bar{u} = u_\infty$ for $y > \delta$ (δ is the thickness of the boundary layer). The dimensions of the small plate along and across the flow (along the x and z axes) are, respectively, L_1 and L_2. It is assumed that there is no transverse (with respect to the flow) flexural of the plate and the edges oriented along the flow are free.

The equation for the deflection of the surface of the plate $y = w(x, t)$ can be written in the form (see [9], p. 94)

$$\gamma \frac{\partial^2 w}{\partial t^2} + D\frac{\partial^4 w}{\partial x^4} + \left[N - \frac{6D}{L_1 h^2}\int_0^{L_1}\left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2} + 2r\frac{\partial w}{\partial t} = -\frac{1}{L_2}\int_{-\frac{1}{2}L_2}^{\frac{1}{2}L_2} p\, dz, \tag{1.1}$$

where γ, h, and D are the surface mass density, the thickness, and the flexural rigidity of the plate; N is the external contraction; and $p(x, z, t)$ is the pressure disturbance on the surface in the flow. We supplement Eq. (1.1) with hinged boundary conditions at the edges: $w = 0$ and $\partial^2 w/\partial x^2 = 0$ at $x = 0, L_1$.

Following [10, 11], we represent the solution of the boundary-value problem for w in the form of a Bubnov–Galerkin series in the system of functions $\sin(mk_0x)$, where $m = 1, 2, \ldots$, $k_0 = \pi/L_1$ is the wave number of the first mode. For not too high flow velocities,

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we consider only the first mode:

$$w = A(t)\sin(k_0 x) \quad (0 < x < L_1). \tag{1.2}$$

Substituting Eq. (1.2) into Eq. (1.1) we arrive at the equation describing the excitation for this mode:

$$\gamma \frac{d^2A}{dt^2} + 2r\frac{dA}{dt} + \left(Dk_0^4 - Nk_0^2\right)A + \frac{3D}{h^2}k_0^4 A^3 = F(t),$$
$$F = -\frac{2}{L_1L_2}\int\limits_0^{L_1}\int\limits_{-\frac{1}{2}L_2}^{\frac{1}{2}L_2} p\sin(k_0x)\,dx\,dz. \tag{1.3}$$

Dropping in Eq. (1.3) for small vibrations the term $\sim A^3$ and setting $(A, p, F) = (A_\omega, p_\omega, F_\omega)\exp(-i\omega t)$, we obtain from Eq. (1.3) the equation

$$\left(-\gamma\omega^2 - 2ir\omega + Dk_0^4 - Nk_0^2\right)A_\omega = F_\omega. \tag{1.4}$$

In order to calculate F_ω we switch to the space-time spectrum of the pressure:

$$\hat{p}(\omega, k_x, k_z) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} p_\omega(x, z)\exp(-ik_xx - ik_zz)\,dx\,dz. \tag{1.5}$$

For a linear medium $\hat{p}$ can be represented as

$$\hat{p}(\omega, k_x, k_z) = Y(\omega, k_x, k_z)\hat{w}(\omega, k_x, k_z), \tag{1.6}$$

where $\hat{w}$ is the frequency-wave spectrum of the displacement of the surface, defined similarly to Eq. (1.5); Y is the complex compressibility of the flow.

We seek Y in the approximation of an incompressible medium. This quantity was found in [6] for two-dimensional deflections of the surface ($k_z = 0$). In the case of three-dimensional deflections an explicit expression for $Y = Y_0$ can be obtained for a uniform flow ($\delta = 0$):

$$Y_0(\omega, k_x, k_z) = -\rho\frac{(\omega - k_xu_\infty)^2}{(k_x^2 + k_z^2)^{1/2}}. \tag{1.7}$$

In the limit $k_x \to 0$ the compressibility of the uniform flow, with respect to two-dimensional deflections, has the asymptotic form $Y_0(\omega, k_x, 0) \to -\rho\omega^2/|k_x|$. The response F can be represented [after inverting the Fourier transform (1.5)] as an integral over k_x with a nonintegrable singularity $\sim -\omega^2/|k_x|$, which gives an infinite apparent mass per unit area of the surface of the plate. It can be shown that this divergence of the apparent mass arises for any monopolar form of the two-dimensional deflection $\left(\int\limits_0^{L_1} w(x, t)\,dx \neq 0\right)$. Thus, excitation of only the first mode (1.2) is possible only for finite L_2.

In the case of a boundary layer of finite thickness $Y \to Y_0$ as $k_x, k_z \to 0$. This can be seen, in particular, from the expression given in [6] for $Y(\omega, k_x, 0)$ for $k_x\delta \ll 1$. Separating the singularity from Y and replacing the remaining smooth function for a small plate oriented across the flow ($\lambda = L_2/L_1 \gg 1$) by its value at the point $k_z = 0$, we obtain

$$Y(\omega, k_x, k_z) \simeq \frac{|k_x|}{(k_x^2 + k_z^2)^{1/2}}Y(\omega, k_x, 0). \tag{1.8}$$

The formula (1.8) expresses the compressibility Y for three-dimensional deflections of the surface through the compressibility, found in [6], with respect to two-dimensional deflections.

Substituting Eqs. (1.6) and (1.8) into Eq. (1.5) we arrive at the problem of calculating an integral of the form

$$\int_{-\infty}^{\infty} \frac{\sin^2 \varkappa}{\varkappa^2} (k_x^2 L_2^2 + 4\varkappa^2)^{-1/2} d\varkappa \left(\varkappa = \frac{1}{2} k_z L_2\right),$$

which can be found approximately by approximating the spectral window $f = \sin^2 \kappa/\kappa^2$ by the function $f = 1/\sqrt{2}$ for $|\kappa| < \pi/\sqrt{2}$ and $f = 0$ for $|\kappa| > \pi/\sqrt{2}$. As a result we obtain an expression for the response of the flow to harmonic vibrations of the plate:

$$F_\omega^{(0)} = -A_\omega \frac{4}{\pi^2} \int_{-\infty}^{\infty} \frac{k_0^3}{(k^2 - k_0^2)^2} \cos^2\left(\frac{\pi k}{2k_0}\right) \sigma\left(\frac{\lambda k}{\sqrt{2}\, k_0}\right) Y(\omega, k, 0)\, dk. \tag{1.9}$$

Here the zero in the superscript denotes the incompressible-fluid approximation; $\sigma(\xi) = (1/2)\xi \ln|(1 + \sqrt{\xi^2 + 1})/(1 - \sqrt{\xi^2 + 1})|$. Since $\sigma \sim \xi \ln \xi$ as $\xi \to 0$, the integral in Eq. (1.9) converges. It can be shown that $F_\omega \sim A_\omega \omega^2 \ln \lambda$ as $\lambda \to \infty$. As one can see from Eq. (1.4), this corresponds to weak divergence of the specific apparent mass for vibrations in the first mode.

In order to employ the algorithm constructed in [6] for calculating the two-dimensional complex compressibility, we switch in Eq. (1.9) to "external" dimensionless variables of the theory of stability of shear flows $k_e = k\delta$, $\bar{k}_0 = k_0\delta$, $\omega_e = \omega\delta/u_\infty$, $Y_e = Y\delta/\rho u_\infty^2$, and we introduce the Reynolds number $R = u_\infty\delta/\nu$, where ν is the kinematic viscosity. Then we write Eq. (1.9) as

$$F_\omega^{(0)} = -\rho u_\infty^2 k_0 G(\omega_e); \tag{1.10}$$

$$G = \frac{4}{\pi^2} \int_0^{\infty} \frac{\bar{k}_0^2}{(k_e^2 - \bar{k}_0^2)^2} \cos^2\left(\frac{\pi k_e}{2\bar{k}_0}\right) \sigma\left(\frac{\lambda k_e}{\sqrt{2}\, \bar{k}_0}\right) \bar{Y}(\omega_e, k_e)\, dk_e, \tag{1.11}$$

where $\bar{Y} = Y_e(\omega_e, k_e) + Y_e^*(-\omega_e, k_e)$; G is, essentially, the dimensionless apparent compressibility for harmonic vibrations of the plate in the first deflection mode. The relation (1.10) can be used to solve the problem of the stability of flexural vibrations in the case when the function G constructed with real values of ω_e is the analytic continuation of the function $G(\omega_e)$ calculated using the formula (1.11) on the contour $\operatorname{Im}\omega_e = \text{const} \to +\infty$ in the complex plane ω_e.† It can be shown that for this the function $\bar{Y}$ should not have any singularities in the half-plane $\operatorname{Im}\omega_e \geq 0$ for real values of k_e (only damped characteristic waves of the flow exist). The function $\bar{Y}(\omega_e, k_e)$ for a turbulent boundary layer for the low values of k_e considered here satisfies this requirement [6].

2. Mutual Apparent Compressibility. Matrix of Apparent Compressibilities for a Uniform Flow. We now consider single-mode vibrations of a pair of identical plates arranged consecutively downstream: $w = A_1 \sin(k_0 x)$ for $0 < x < L_1$, $w = A_2 \sin[k_0(x - L_1)]$ for $L_1 < x < 2L_1$, and $w = 0$ for $x < 0$ and $x > 2L_1$. The procedure for calculating the response $F_\omega^{(0)}$ in this case is completely analogous to the procedure constructed in Sec. 1. For this reason, we present without derivation the extension of the formula (1.10) for two plates:

$$F_{1,2}^{(0)} = -\rho u_\infty^2 k_0 (G A_{1,2\omega} + G_{1,2} A_{2,1\omega}). \tag{2.1}$$

Here $F_j^{(0)}$ is the right-hand side of Eq. (1.3), in which we made the substitution $A \to A_j$ ($j = 1, 2$); $G_{1,2}$ are calculated using the formula (1.11) but with $\bar{Y}$ defined differently:

$$\bar{Y} \to \bar{Y}_{1,2} = Y_e(\omega_e, k_e)\, e^{\mp i\pi(k_e/\bar{k}_0)} + Y_e^*(-\omega_e, k_e)\, e^{\pm i\pi(k_e/\bar{k}_0)}. \tag{2.2}$$

†This follows from the procedure employed for solving the problem of the response of the flow by the Laplace transform method.

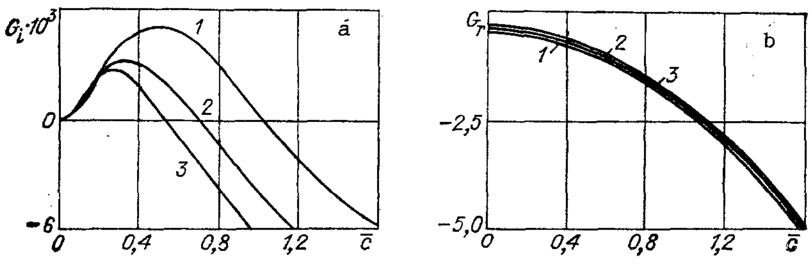

Fig. 1

In accordance with Eq. (2.1) the quantities G and $G_{1,2}$ are elements of the dimensionless matrix G_{ij} of the apparent compressibilities for a pair of plates, and $G_{11} = G_{22} = G$, $G_{12} = G_1$, $G_{21} = G_2$.

It is not difficult to find explicitly the matrix of apparent compressibilities in the case of a uniform potential flow over the plates. Using Eqs. (1.7) and (1.9) we obtain

$$G^{(p)} = -(a_0\bar{c}^2 + d_0), \tag{2.3}$$

where $\bar{c} = \omega_e/\bar{k}_0$; the coefficients a_0 and d_0, determining the effects of the apparent mass and static elasticity, have the form

$$a_0 = \int\limits_{-\infty}^{\infty} \frac{8}{\pi^2\varkappa(1-\varkappa^2)^2}\cos^2\left(\frac{\pi}{2}\varkappa\right)\sigma\left(\frac{\varkappa\lambda}{\sqrt{2}}\right)d\varkappa,$$
$$d_0 = \int\limits_{-\infty}^{\infty} \frac{8\varkappa}{\pi^2(1-\varkappa^2)^2}\cos^2\left(\frac{\pi}{2}\varkappa\right)\sigma\left(\frac{\varkappa\lambda}{\sqrt{2}}\right)d\varkappa \tag{2.4}$$

and depend only on λ. For $\lambda = 3$ the calculations give $a_0 = 1.8$ and $d_0 = 0.71$. The off-diagonal elements of the matrix of apparent compressibilities are found similarly:

$$G_{1,2}^{(p)} = -(a_1\bar{c}^2 - ib_1\bar{c} + d_1). \tag{2.5}$$

The expressions for a_1, b_1, and d_1 have a structure similar to the expressions (2.4). For $\lambda = 3$, $a_1 = 0.54$, $b_1 = -0.45$, and $d_1 = -0.14$. In what follows, all numerical calculations are also performed for $\lambda = 3$.

3. Calculation of the Matrix of Apparent Compressibilities for the Average Flow in a Turbulent Boundary Layer. The calculations were performed using the formulas (1.10), (1.11), and (2.2). The one-dimensional complex compressibility was found by the method of [6]. The integral in Eq. (1.11) was calculated using 40 points and an upper limit of $3\bar{k}_0$.

Figures 1a and b display the imaginary and real parts of the function $G = G_r + iG_i$ with $R = 8\cdot10^4$ (the curves 1-3 correspond to $\bar{k}_0 = 1; 2; 3$). It can be shown that for $G_i > 0$ the energy flux is directed toward the surface (similarly to [6]). The calculations show that the quasipotential approximation is applicable for G to a high degree of accuracy: $G = G^{(p)}$ + small complex correction, and in the expression (2.3) for $G^{(p)}$ the coefficient d_0 depends on $\bar{k}_0$ and R (for a uniform flow $\bar{k}_0 \to 0$). In particular, for $R = 8\cdot10^4$, $d_0 = 0.46; 0.38; 0.34$ for $\bar{k}_0 = 1; 2; 3$, respectively.

The degree to which G_i depends on R with $\bar{k}_0 = 1$ is illustrated in Fig. 2 (the curves 1-3 correspond to $R = 4\cdot10^4; 8\cdot10^4; 1.5\cdot10^5$). The coefficient d_0 does not change significantly in this range of values of R.

The results of calculating $G_{1,2r}$ and $G_{1,2i}$ with $\bar{k}_0 = 1$, $R = 8\cdot10^4$ are presented in Fig. 3a. The quasipotential approximation is also applicable to $G_{1,2} = G_{1,2}^{(p)}$ + small complex corrections, where in the expression for (2.4) for $G_{1,2}^{(p)}$ the coefficients b_1 and d_1 depend on $\bar{k}_0$ and R. In particular, for $\bar{k}_0 = 1$, $R = 8\cdot10^4$ $b_1 = -0.43$, $d_1 = -0.10$, and b_1 and d_1 do not vary significantly in the indicated range of values of R. The energy flux to the surface, due to interaction of the plates, is determined by the nonzero off-diagonal elements

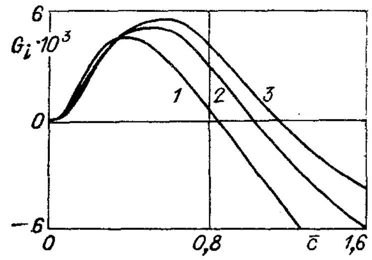

Fig. 2

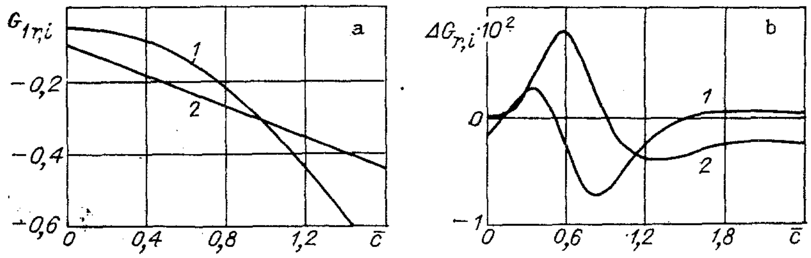

Fig. 3

of the anti-Hermitian part of the matrix G_{ij}, i.e., the difference $\Delta G = G_1^* - G_2$. The dependence of the real and imaginary parts of ΔG on c with $\bar{k}_0 = 1$, $R = 8 \cdot 10^4$ is shown in Fig. 3b (ΔG_r, ΔG_i – lines 1, 2).

We underscore the fact that the physically graphic quasipotential model for G_{ij} is based on the results of numerical calculations and can be considered only as an approximation of G_{ij} in a finite frequency band.

4. Taking into Account Losses Due to Emission of Sound. In a medium with finite compressibility, energy losses associated with emission of sound appear. We confine our attention to acoustically compact plates ($\omega L_{1,2}/c_a \ll 1$, c_a is the speed of sound), when for $L_2 \gg L_1$ the hydrodynamic disturbances in the boundary layer (whose thickness satisfies the condition $\omega\delta/c_a \ll 1$) are quasi-two-dimensional but the acoustic field is not. Regarding the medium as stationary and using a well-known expression from the theory of radiation for the pressure field from a radiator "in the screen" [12], it is possible to calculate the first correction in the expansion of the pressure with respect to the delay time of the sound within the plate. This component of the pressure determines the acoustical component of the response of the medium $F_{1,2}^{(a)}$ [see Eq. (1.4)]. For a surface consisting of two plates we obtain the expression

$$F_{1,2}^{(a)} = \frac{4\rho L_1 L_2}{\pi^3 c_a}\left(\frac{d^3 A_1}{dt^3} + \frac{d^3 A_2}{dt^3}\right). \tag{4.1}$$

The formula (4.1) can also be used in the case when a flow with low Mach number ($M = u_\infty/c_a \ll 1$) is present in the region $y > 0$. Since the pseudosonic component of the pressure also does not change much for $M \ll 1$, we represent the total response in the form $F_{1,2} = F_{1,2}^{(0)} + F_{1,2}^{(a)}$, where $F_{1,2}^{(0)}$ is the response, found in Secs. 1 and 2, for an incompressible medium.

We note that for strictly antiphase motion of the plates ($A_2 = -A_1$) the monopolar component of the radiation losses (4.1) vanishes. It can be shown that the expression (4.1) is correct when the difference of the amplitudes of the vibrations of the plates is sufficiently large: $|(A_{2\omega} - A_{1\omega})/A_{1,2\omega}| \gg L_2\omega/c_a$.

5. Instability Increment of Flexural Vibrations. In order to study the characteristic vibrations we introduce, following [6], the new dimensionless variables $\bar{\omega} = \omega/\omega_0$, $V = u \propto k_0/\omega$ ($\omega_0 = [(Dk_0^4 - Nk_0^2)/\gamma_0]^{1/2}$, $\gamma_0 = \gamma + \rho/k_0$ (also $\bar{c} = \bar{\omega}/V$)), as well as the coefficients $\alpha = \gamma/\gamma_0$, $\alpha_1 = 1 - \alpha$ and the Reynolds number with respect to the phase velocity of the flexural waves $R_0 = \omega_0\delta/k_0\nu$ (correspondingly, $R = R_0 V$).

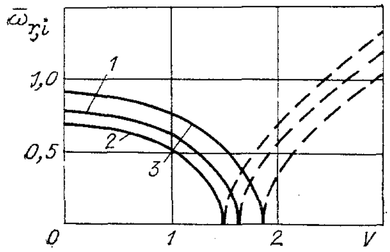

Fig. 4

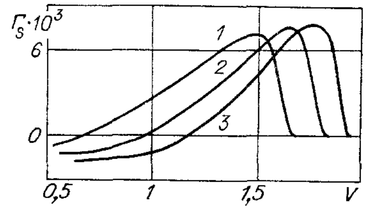

Fig. 5

We consider first the vibrations of a single plate. Using the quasipotential approximation (see Sec. 3), we rewrite Eq. (1.4) in the form

$$m_0\bar{\omega}^2 + \alpha_1 V^2 d_0 - 1 = \alpha_1 V^2 (G - G^{(p)}) - i\xi\bar{\omega}^3 - 2i\bar{r}\bar{\omega}, \qquad (5.1)$$

where $m_0 = \alpha + \alpha_1 a_0$; $\xi = (4/\pi^2)\alpha_1 \omega L_2/c_a \ll 1$; $\bar{r} = r/\gamma_0\omega_0$. We express the coefficient $\bar{r}$ in terms of the quality factor Q_f of a free (placed in a vacuum) plate: $\bar{r} = (1/2Q_f)\sqrt{\alpha_1/(1+\alpha_1)}$.

In the absence of dissipation [the right-hand side in Eq. (5.1) is zero] we obtain the frequency $\bar{\omega}^{(0)} = [(1 - \alpha_1 V^2 d_0)/m_0]^{1/2}$, which becomes imaginary when $V > V_c = 1/\sqrt{\alpha_1 d_0}$ (V_c is the critical divergence velocity of the plate in a quasipotential flow). The dependence of the real and imaginary parts of the frequency $\bar{\omega}^{(0)} = \bar{\omega}_r + i\bar{\omega}_i$ on V for $\alpha = 0.2$ ($\alpha_1 = 0.8$), $\bar{k}_0 = 1$, and $R_0 = 8\cdot 10^4$ is shown by curve 1 in Fig. 4 (solid lines – $\bar{\omega}_r$, dashed lines – $\bar{\omega}_i$). We note that the computational results for $\alpha \ll 1$ (under conditions when the apparent mass is greater than the characteristic mass) are virtually independent of α. In what follows, all calculations are performed with $\alpha = 0.2$. The curves 2 and 3 show the behavior of the characteristic frequencies in the similar problem for a pair of plates. The magnitude of the shift between them indicates that the coupling of the plates is weak.

In the subcritical region ($V < V_c$) Eq. (5.1) can be solved by the method of perturbation with respect to the small quantities $|(G - G^{(p)})|$, ξ, $\bar{r}$ (similarly to [1]). The calculations lead to the following expression for the increment:

$$\bar{\omega}_i \approx \frac{1}{m_0}\left[\frac{\alpha_1 V^2}{2\bar{\omega}^{(0)}}(G_i)_* - \frac{1}{2}\xi\bar{\omega}^{(0)^2} - \bar{r}\right] \qquad (5.2)$$

where the index * indicates that the quantity is evaluated at $\bar{c} = \bar{\omega}^{(0)}/V$). Since $\bar{\omega}^{(0)} \to 0$ as $V \to V_c$, the quantity G_i becomes positive in the subcritical region. The instability arising in this case ($\bar{\omega}_i > 0$) is due to the irreversible extraction of energy from the central flow into the boundary layer. The results of the calculations of the aerodynamic part of the increment Γ_s [the first term in Eq. (5.2)] for $V < V_c$, $R_0 = 8\cdot 10^4$ are presented in Fig. 5 (curves 1-3 correspond to $\bar{k}_0 = 1;\ 2;\ 3$). In this calculation the dependence of G_i on R was neglected (it was assumed that $R = R_0$). The plots in Fig. 1 can be used to construct the boundary of the region where subcritical instability is suppressed in the $\bar{r}$, ξ plane or to write down the condition under which it is stable for a fixed value of V. For example, for $\bar{k}_0 = 1$ and $V = 1.12$ (at the maximum of $G_i(\bar{c})$ – Fig. 1a) stabilization occurs for $\bar{r} + 0.16\xi > 0.0045$, while for $\bar{k}_0 = 1$, $V = 1.5$ [at the maximum of $\Gamma_s(V)$] we obtain $\bar{r} + 0.052\xi > 0.0082$.

Subcritical instability of a pair of plates is calculated similarly. The computational results for the matrix of apparent compressibilities also make it possible to study the effect of the coupling between the plates on instability in a long chain of plates extending along the flow. Neglecting radiation losses in this case, we obtain an equation for the amplitude $\hat{A}_n$ of the n-th deflection of the plate in the form

$$(\alpha\bar{\omega}^2 + 2i\bar{r}\bar{\omega} - 1)\hat{A}_n = \alpha_1 V^2 (G\hat{A}_n + G_1\hat{A}_{n+1} + G_2\hat{A}_{n-1}). \qquad (5.3)$$

The simplest and very characteristic case to analyze (see [6]) is the case of periodic sign-alternating deflection in an infinite chain of plates, when $\hat{A}_n = (-1)^n\hat{A}_0$. We represent the

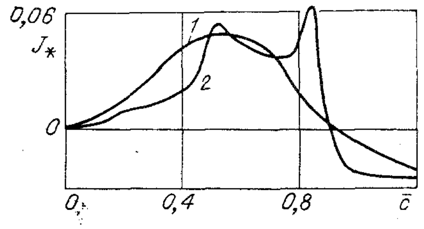

Fig. 6

expression for the increment of the subcritical instability of such a deflection as

$$\bar{\omega}_i \simeq -\frac{\bar{r}}{m_{01}} + \frac{\alpha_1 V^2}{4\bar{\omega}^{(0)}} J_* \left[J_* = \frac{2}{m_{01}}(G_i + \Delta G_i)_*\right], \quad (5.4)$$

where $m_{01} = \alpha + \alpha_1(a_0 - 2a_1)$; $\bar{\omega}^{(0)} = [(1 - \alpha_1 d_0 V^2)/m_{01}]^{1/2}$. A similar expression is obtained in [6] for the increment in an infinite chain. This expression actually takes into account all couplings between plates. It differs from Eq. (5.4) in that the definitions of J_* and $\bar{\omega}^{(0)}$ are different, and by the fact that it does not contain the coefficient $1/m_{01}$ in front of $\bar{r}$. The latter difference is evidently caused by the fact that the coupling coefficients between the plates via the apparent mass (a_0, a_1, etc.) are not summed completely. The difference in $\bar{\omega}^{(0)}$ and the critical divergence velocities in these two cases is small (~5%). Figure 6 shows the computational results for J_* with $\bar{k}_0 = 1$, $R = 8 \cdot 10^4$ (curve 1) and the analog of this quantity from [6] (curve 2). Comparing them shows that the increments obtained taking into account the couplings partially and completely agree satisfactorily with one another.

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